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On the strong law of large numbers for some stochastic processes

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We present generalizations of some results obtained in [1] concerning the strong law of large numbers (*SLLN*) for stochastic processes. The basic definition of the *SLLN* can be found in [2].

Definition [2]. Assume that $\{Y_t\}_{t=0}^{\infty}$ and $\{L_t\}_{t=0}^{\infty}$ with $L_0 = 0$ are a semimartingale and a predictable increasing process defined on a stochastic basis $\{\Omega, \mathcal{F}, \mathbf{F} = (F_t)_{t\geq 0}, \mathbf{P}\}$. We say that the pair (Y_t, L_t) satisfies the *SLLN* if $P(\lim\limits_{t\to\infty}\{Y_t/L_t\} = 0) = 1$.

Our main purpose is to establish the *SLLN* when Y_t is a function of an n-dimensional random process $\{X_t\}_{t=0}^{\infty}$ given by

$$dX_t = A_t X_t \, dt + G_t \, dw_t \,, \quad X_0 = x \,, \tag{1}$$

where $A_t \in R^{n\times n}$, $G_t \in R^{n\times d}$ are bounded non-random matrix functions, $\{w_t\}_{t=0}^{\infty}$ is a d-dimensional standard Wiener process and x is a non-random vector. We make the following assumption.

Assumption $\mathcal{A}$. There exist positive constants κ_1, κ_2 such that $\|\Phi(t,s)\| \leq \kappa_1 e^{-\kappa_2(t-s)}$, for all $s \leq t$, where $\Phi(t,s)$ is the fundamental matrix corresponding to A_t, $\|\cdot\|$ denotes the Euclidean matrix norm.

The main result is the following theorem.

Theorem. Let $L_t = \int\limits_0^t \|G_s\|^2 \, ds$, $Y_t = \|X_t\|^2$, where X_t is a solution of (1). Assuming $\mathcal{A}$, the pair (Y_t, L_t) satisfies the *SLLN*.

We also prove some auxiliary statements imposing conditions on L_t in order to ensure that the *SLLN* holds for the pair (Y_t, L_t), when Y_t could be the Ito or Riemann-type integral of a rather general stochastic process. The methods used in the proofs are similar to those applied in [3].

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Литература

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